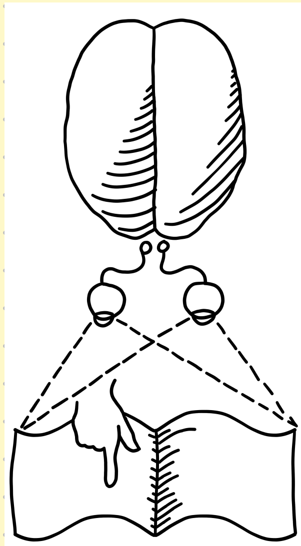
CS4510 6-1 The Church-Turing Thesis

Defn for any algorithm, computation process, mechanical procedure
∃ an equivalent Turing Machine

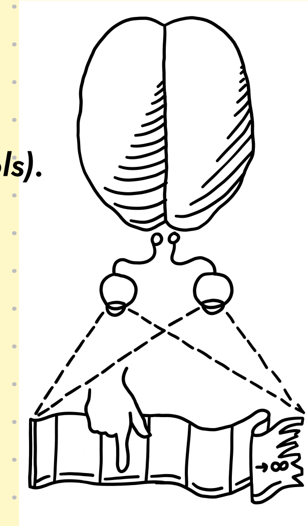
Church "effective calcuability" λ 1936
Turing " computable " TMs

The Direct Appeal to Intuition (On computable Numbers)



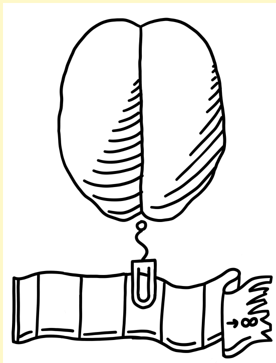
Computing is normally done by writing certain symbols on paper. We may suppose this paper is divided into squares like a child's arithmetic book. In elementary arithmetic the two-dimensional character of the paper is sometimes used. But such a use is always avoidable, and I think that it will be agreed that the two-dimensional character of paper is no essential of computation. I assume then that the computation is carried out on one-dimensional paper, i.e. on a tape divided into squares.

I shall also suppose that the number of symbols which may be printed is finite. If we were to allow an infinity of symbols, then there would be symbols differing to an arbitrarily small extent†. The effect of this restriction of the number of symbols is not very serious. It is always possible to use sequences of symbols in the place of single symbols. Thus an Arabic numeral such as 17 or 999999999999999 is normally treated as a single symbol. Similarly in any European language words are treated as single symbols (Chinese, however, attempts to have an enumerable infinity of symbols).

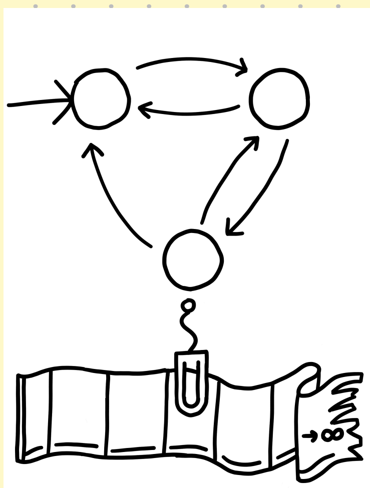


The behaviour of the computer at any moment is determined by the symbols which he is observing, and his "state of mind" at that moment. We may suppose that there is a bound B to the number of symbols or squares which the computer can observe at one moment. If he wishes to observe more, he must use successive observations.

We will also suppose that the number of states of mind which need be taken into account is finite. The reasons for this are of the same character as those which restrict the number of symbols. If we admitted an infinity of states of mind, some of them will be "arbitrarily close" and will be confused. Again, the restriction is not one which seriously affects computation, since the use of more complicated states of mind can be avoided by writing more symbols on the tape.



We may now construct a machine to do the work of this computer. To each state of mind of the computer corresponds an "m-configuration" of the machine. The machine scans B squares corresponding to the B squares observed by the computer. In any move the machine can change a symbol on a scanned square or can change any one of the scanned squares to another square distant not more than L squares from one of the other scanned squares. The move which is done, and the succeeding configuration, are determined by the scanned symbol and the m-configuration



if M is any kind of "computing machine" then

$L(M) \subseteq L(TM)$

There does not exist a "computing machine" M such that

$L(TM) \subsetneq L(M)$

Defn an encoding (Godel Numbering) is a string representation of any mathematical object.

The encoding of object G is denoted by $\langle G \rangle$

$\langle G_1, G_2 \rangle$ $\langle M \rangle$

Example 1 There are infinitely many Turing Machines.

Let $\{f_i\}_{i=1}^{\infty}$ be a family of constant functions.

$\forall x \quad f_i(x) = i$

by the CTT $\exists$ a TM for each f_i

Example 2 $\exists$ a TM to find roots of a polynomial

Since you can come up with an algorithm, by CTT, there must exist a TM

Example 3 $\exists$ a TM UTM which takes input $\langle M, w \rangle$ and simulates M on w.