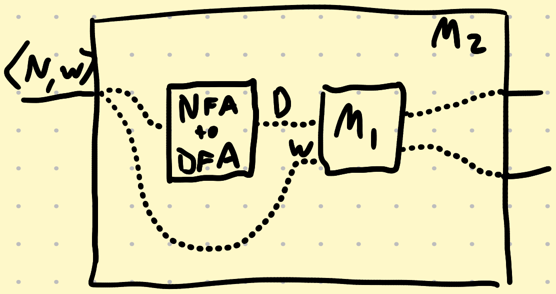
CS4510 8-1 Some Decidable Languages

$A_{DFA} = \{ \langle D, w \rangle \mid D(w) \text{ accept} \}$ is decidable.

$D(w)$ runs in exact $|w|$ steps. Create a decider M_1 of A_{DFA} which simulates w on D for exactly $|w|$ steps. If $D(w)$ accepts, then accept. If $D(w)$ rejects, then reject.

$A_{NFA} = \{ \langle N, w \rangle \mid N(w) \text{ accepts} \}$ is decidable

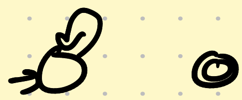
Let M_2 be our decider for A_{NFA}. M_2 converts N to DFA D. then runs M_1 on $\langle D, w \rangle$ and returns what M_1 returns.



Similarly $A_{REX} = \{ \langle R, w \rangle \mid R \text{ generates } w \}$

$A_{RG} = \{ \langle G, w \rangle \mid G \text{ generates } w \}$

$E_{DFA} = \{ \langle D \rangle \mid L(D) = \emptyset \}$ is decidable

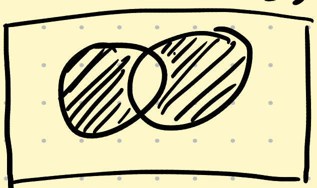


mark start state
while no new states are marked
    for each state s
        if state has incoming arrow from marked state
            mark s
if no final state is marked, accept $\langle D \rangle$

$EQ_{DFA} = \{ \langle D_1, D_2 \rangle \mid L(D_1) = L(D_2) \}$ is decidable

$L(D_1) \triangle L(D_2) =$

$L(D_1) \cup L(D_2) - (L(D_1) \cap L(D_2))$



$= (L(D_1) \cap \overline{L(D_2)}) \cup (\overline{L(D_1)} \cap L(D_2))$

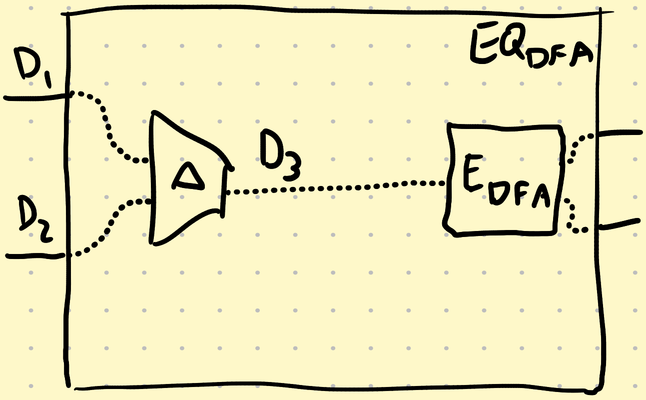
$L(D_1) \triangle L(D_2) = \emptyset \iff L(D_1) = L(D_2)$

decider for EQ_{DFA} =

construct DFA D_3 such that $L(D_3) = L(D_1) \triangle L(D_2)$

Run D_3 on decider of E_{DFA}

return w/e E_{DFA} decider returns.



$E_{CFG} = \{\langle G \rangle \mid L(G) = \emptyset\}$
$(S \to S;\ \ S \to A,\ A \to S)$
is decidable

mark all terminals
while no new nonterminals are marked
  if $A \to$ (a string of <u>all</u> marked)
    mark A
Accept iff start symbol <u>not</u> marked.

Every CFG is decidable

1 each CFL has a PDA, simulate a PDA on a TM tape

2 use decider for A_{CFG}.

M_3 on input w, run decider $\langle G, w \rangle$ and accept iff the decider accepts

$A_{CFG} = \{\langle G, w \rangle \mid G \text{ generates } w\}$ is decidable. Recall CNF. If G is in CNF, $|w| = n$, w take $2n-1$ derivations/productions

1 Convert G to CNF Form
produce a list of all words of length $|w|$ called W
accept iff $w \in W$.

EQ_{CFG} is undecidable!