CS4510 9-2 (PCP) Post Correspondence Problem

we have a set of dominoes like $\left\{ \left[\frac{t_1}{b_1}\right], \left[\frac{t_2}{b_2}\right], \ldots, \left[\frac{t_k}{b_k}\right] \right\}$

where each $t_i, b_i \in \Sigma^*$

$$\left[\frac{a}{ab}\right]\left[\frac{ba}{a}\right] = \frac{aba}{aba}$$

for each $|t_i| > |b_j|$
or if unary

$PCP = \{ \langle P \rangle \mid P \text{ has a match} \}$

$MPCP = \{ \langle P \rangle \mid P \text{ has a match with the first domino starting} \}$

$\cdot u = \cdot u_1 \cdot u_2 \cdot u_3 \cdots \cdot u_n$ $\qquad u = u_1 \ldots u_n$

$u \cdot = u_1 \cdot u_2 \cdot u_3 \cdot \ldots \cdot u_n \cdot$

$\cdot u \cdot = \cdot u_1 \cdot \ldots \cdot u_n \cdot$ $\qquad$ If we have set like

$$\left\{ \begin{bmatrix} t_1 \\ b_1 \end{bmatrix} \begin{bmatrix} t_2 \\ b_2 \end{bmatrix} \cdots \begin{bmatrix} t_k \\ b_k \end{bmatrix} \right\}$$

$$\left\{ \begin{bmatrix} \cdot t_1 \\ \cdot b_1 \cdot \end{bmatrix} \begin{bmatrix} \cdot t_2 \\ b_2 \cdot \end{bmatrix} \cdots \begin{bmatrix} \cdot t_k \\ b_k \cdot \end{bmatrix} \begin{bmatrix} \cdot \square \\ \square \end{bmatrix} \right\}$$

How to make P' from M, w

1) set $\begin{bmatrix} t_1 \\ b_1 \end{bmatrix} = \begin{bmatrix} \# \\ \# q_0 w_1 w_2 \ldots w_n \# \end{bmatrix}$

2) if $a, b \in \Gamma$ $\quad q_i, q_j \in Q$ $\quad \delta(q_i, a) = (q_j, b, R)$

add tile $\begin{bmatrix} t_i \\ b_i \end{bmatrix}$ $\begin{bmatrix} q_i a \\ b q_j \end{bmatrix}$ $\qquad q_i \neq q_{reject}$

3) if $a, b, c \in \Gamma$ $\quad q_i\ q_j \in Q$ $\quad \delta(q_i, a) = (q_j, b, L)$

add tile $\begin{bmatrix} t_i \\ b_i \end{bmatrix}$ $\begin{bmatrix} c q_i a \\ q_j c b \end{bmatrix}$

4) $\forall a \in \Gamma$ add tile $\begin{bmatrix} a \\ a \end{bmatrix}$

5) add tiles $\begin{bmatrix} \# \\ \# \end{bmatrix}$ and $\begin{bmatrix} \# \\ \sqcup \# \end{bmatrix}$

6) add tiles $\forall a \in \Gamma$ $\begin{bmatrix} a\, q_a \\ q_a \end{bmatrix}$ and $\begin{bmatrix} q_a\, a \\ q_a \end{bmatrix}$

7 add tile $\begin{bmatrix} q_a \# \# \\ \# \end{bmatrix}$

$\delta(q_0, 1) = (q_1, 0, R)$ $\quad C_0 = q_0 101$

$\begin{bmatrix} q_0 1 \\ 0 q_1 \end{bmatrix}$ $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$\begin{bmatrix} \# \\ \# q_0 101 \# \end{bmatrix} \begin{bmatrix} q_0 1 \\ 0 q_1 \end{bmatrix}$

$\# q_0 1$

$\# q_0 101 \# 0 q_1$

$\# C_0 \# \dots \# C_i \boxed{}$

$\# C_0 \# \dots \# C_i \# C_{i+1} \boxed{}$

on input $\langle M, w \rangle$

construct P' from M, w

construct P from P'

if $P \in PCP$ (P has a match

accept

else

reject

Hillbert 10^{th} problem is there an algorithm to decide if a diophantine equation has integer solution

$3x^2 + xyz - y^3 = 0$

$x^2 - 1 = 0 \quad \sqrt{2}$

in MTG if one player has advantage or not.